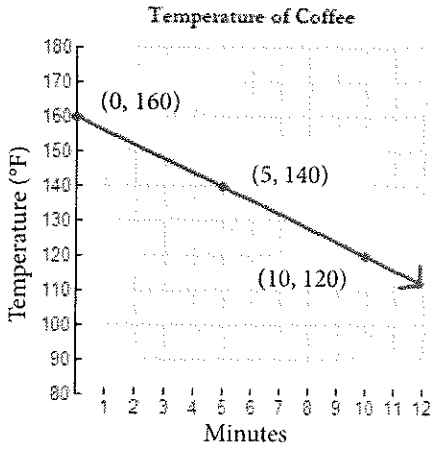
WRITING LINEAR EQUATIONS FOR GRAPHS

LESSON 3.2

Write a linear equation for a given graph.

EXPLORE! **FIND THE EQUATION**

Stacey and Mario like to go to the coffee shop before school. They decided to conduct an experiment to study the rate at which their coffees cool when left untouched on the table. The graph below shows the information they gathered.



Step 1: What is the real world meaning of the point (0, 160)? How about the point (10, 120)?

Step 2: Use the slope formula, $\frac{y_2 - y_1}{x_2 - x_1}$, to find the slope of the line. Does it matter which points from the graph you use in the formula?

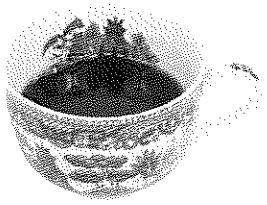
Step 3: What is the real-world meaning of the slope?

Step 4: What is the y-intercept of this graph?

Step 5: Write an equation in slope-intercept form that represents this graph.

Step 6: Use your equation to determine the temperature of the coffee after 12 minutes.

Step 7: According to Stacey and Mario's experiment, the coffee continued to cool at the same rate every minute that passed. Do you think the coffee will continue to cool at this rate if the coffee is left on the table for one hour? Verify your theory using your equation.



A linear equation can be written for a specific line if you know the slope and y-intercept. The y-intercept can be determined by locating the point where the graph crosses the y-axis $(0, b)$. The slope must be calculated using a slope triangle or the slope formula. Remember that when dealing with real-world graphs, the y-intercept is referred to as the start value and the slope is called the rate of change.

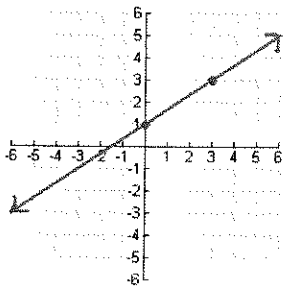
WRITING A LINEAR EQUATION FROM A GRAPH

1. Locate the y-intercept on the graph.
2. Find the slope of the line.
3. Write the equation in slope-intercept form, $y = mx + b$.

EXAMPLE 1

Determine the slope and y-intercept of each graph. Write the equation for each graph in slope-intercept form.

a.



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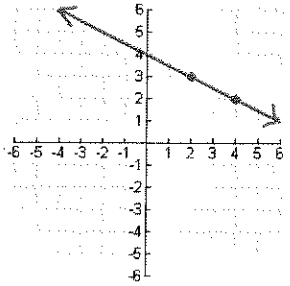

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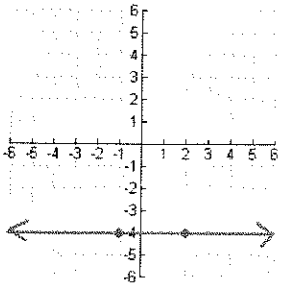
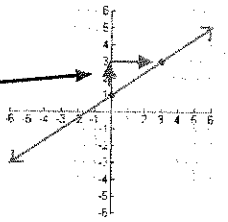


SOLUTIONS

a. The line crosses the y-axis at 1 so $b = 1$.
The slope triangle shows that $m = \frac{2}{3}$.
The equation in slope-intercept form is: $y = \frac{2}{3}x + 1$.



b. The line crosses the y-axis at 4 so $b = 4$.
The slope triangle shows that $m = -\frac{1}{2}$.
The equation in slope-intercept form is: $y = -\frac{1}{2}x + 4$.

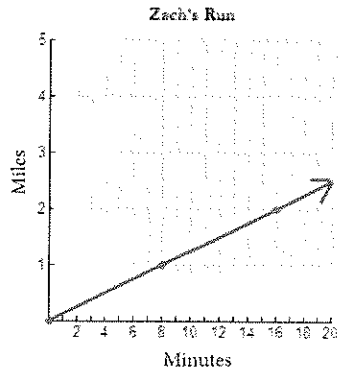
c. The line crosses the y-axis at -4 so $b = -4$.
The slope triangle shows that $m = \frac{0}{3} = 0$.
The equation in slope-intercept form is $y = 0x - 4$ which can also be written as $y = -4$.

It is very useful to have equations for graphs that represent real-world situations because you can use the equation to predict future or past data.

EXAMPLE 2

Zach enjoys running each day after school. The graph below represents the distance Zach has traveled based on the number of minutes he has been running.

a. Find the slope-intercept equation that represents the situation shown on the graph.
b. Use your equation to determine how far Zach will have gone in 28 minutes.
c. Use your equation to determine how long it will take Zach to run 10 miles.



SOLUTIONS

a. The line crosses the y-axis at 0 so $b = 0$.
The two marked points are (8, 1) and (16, 2). Use the slope formula to calculate the slope.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 1}{16 - 8} = \frac{1}{8}$$

The slope-intercept equation is $y = \frac{1}{8}x + 0$ or $y = \frac{1}{8}x$.

EXAMPLE 2
SOLUTIONS
(CONTINUED)

b. Since the x-values represent minutes, substitute 28 for x to determine how far Zach runs in 28 minutes:

$$y = \tfrac{1}{8}x$$
$$y = \tfrac{1}{8}(28)$$
$$y = \tfrac{28}{8} = 3\tfrac{4}{8} = 3\tfrac{1}{2} = 3.5$$

Zach runs 3.5 miles in 28 minutes.

c. Since the y-values represent miles, substitute 10 for y to determine how long it will take Zach to run 10 miles.

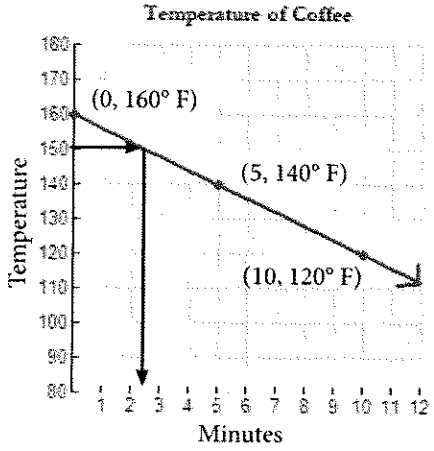
$$10 = \tfrac{1}{8}x$$
$$\tfrac{8}{1} \cdot 10 = \tfrac{1}{8}x \cdot \tfrac{8}{1}$$
$$80 = x$$

Multiply by the reciprocal.

Zach's 10 mile run will take 80 minutes (one hour and twenty minutes).

Stacey and Mario found it was easy to write a slope-intercept equation from the graph because all they had to do was find the y-intercept and slope and put it into the form $y = mx + b$. They found the equation for their coffee experiment in the Explore! to be $y = -4x + 160$. They tested their formula by substituting values for the temperature of the coffee to see if it matched the number of minutes shown on the graph.

Temperature = 150° F → Since the y-values represent the temperature, substitute 150 for y.



$$150 = -4x + 160$$
$$-160 \quad\quad -160$$
$$-10 = -4x$$
$$-4 \quad\quad -4$$
$$2.5 = x$$

The coffee was 150° after 2.5 minutes.

Stacey and Mario used their formula to predict when their coffee would freeze.
Temperature = 32° F (freezing) → Since the y-values represent temperature, substitute 32 for y.

$$32 = -4x + 160$$
$$-160 \quad\quad -160$$
$$-128 = -4x$$
$$-4 \quad\quad -4$$
$$32 = x$$

According to the equation, the coffee would freeze in 32 minutes.

Stacey and Mario decided their formula only works for the first 20 minutes or so. It is not likely that coffee is going to reach a freezing temperature while it is sitting on the table.

EXERCISES

Identify the slope and y-intercept of each graph. Write the corresponding linear equation in slope-intercept form.

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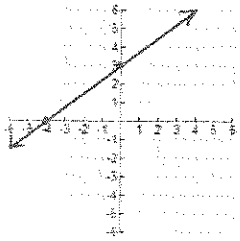

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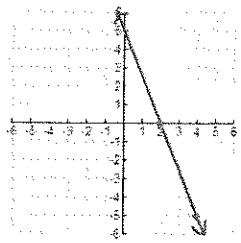

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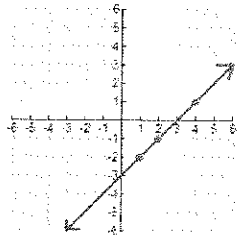

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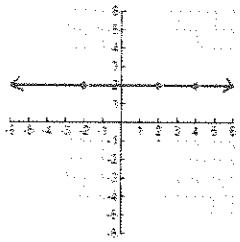

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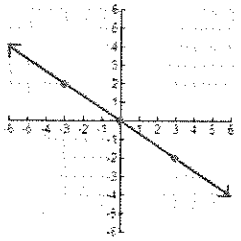

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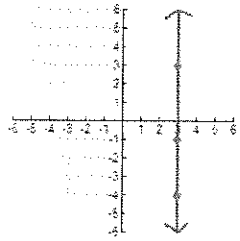
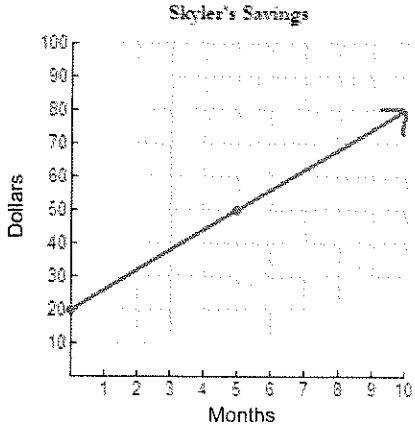


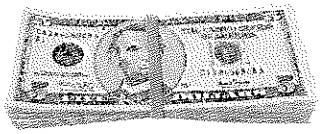
7. Skyler's total savings are shown on the graph.

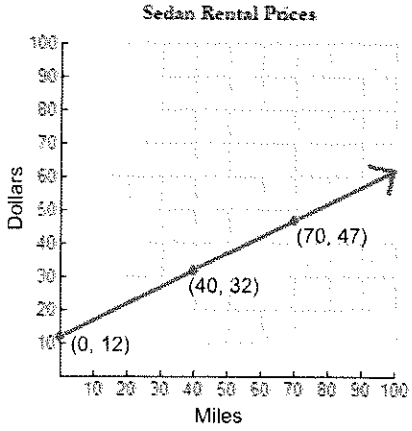
a. Find the slope-intercept equation that represents the graph.

b. Determine how much Skyler will have in his savings account after 18 months.

c. Determine how many months it will take before Skyler has $212 in his savings. Show all work necessary to justify your answer.







8. Javier owns a car rental company. He provides the graph seen at left for his customers to see the price for renting a sedan based on the number of miles they drive.

a. Find the equation (in slope-intercept form) that represents the amount Javier charges based on the number of miles driven.

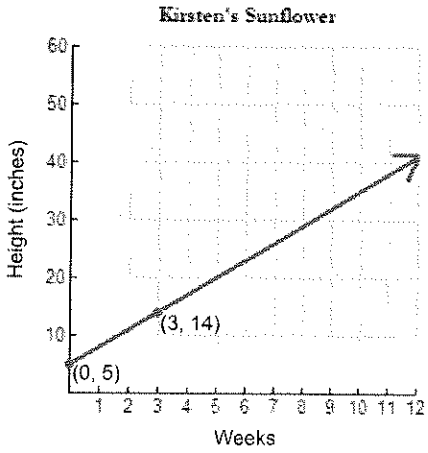
b. Determine the amount a customer will have to pay if she rents a sedan and drives it 120 miles.

c. Leticia rented a sedan from Javier. When she returned it, her bill was $38.50. How many miles did she drive? Show all work necessary to justify your answer.

9. What is the minimum number of points needed on a linear graph to find the equation for the line? Explain your reasoning.

10. What type of linear equation does not have a y-intercept? Give an example of a graph and its corresponding equation that fit this category of linear equations.

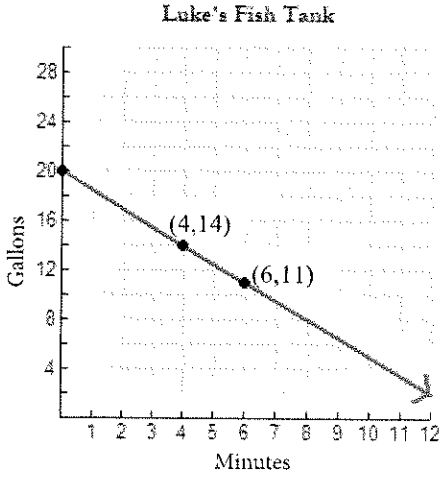
11. At two different times during the summer, Kirsten measured the height of a sunflower she had planted in May. She measured it when she first planted the flower and then again 3 weeks later.

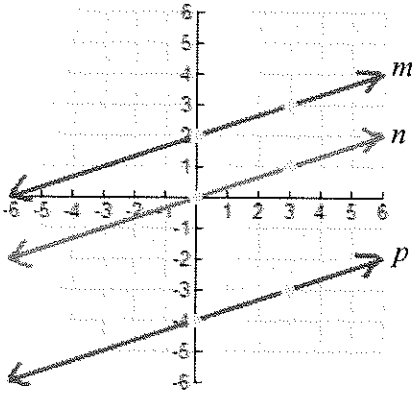


a. Find the slope-intercept equation that represents the height of Kirsten's flower based on the number of weeks since she planted it.
b. Determine exactly how tall the sunflower will be after 8 weeks.
c. Determine how many weeks have passed if the plant is 47 inches tall.

12. Luke drained his 20-gallon fish tank. At two different times, he measured the amount of water left in the tank. He graphed the information on the graph shown at right.

a. Find the slope-intercept equation that represents the number of gallons left in the fish tank since he began draining it.
b. Use your equation to determine how much water will be left in the tank after 10 minutes.
c. When will the water be completely drained from the tank? Show all work necessary to justify your answer.





13. Use the graph at left to answer the following questions.

a. Find the slope-intercept equations for lines m, n and p.
b. What do the three equations have in common?
c. What geometry term can be used to describe the relationship between these three lines?

REVIEW

14. Evaluate the following expressions when $x = 3$ and $y = -4$.

a. $2x + 5y$ **b.** $-4x - 6y$ **c.** $\frac{1}{2}y - 5x$

State whether each equation is true or false for the values of the variables given.

15. $y = 3x + 1$ when $x = 2$ and $y = 6$

16. $y = \frac{4}{3}x - 4$ when $x = 6$ and $y = 4$

17. $y = -2x + 7$ when $x = 5$ and $y = -3$

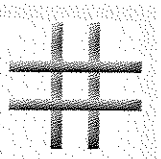
18. $y = \frac{1}{4}x$ when $x = 10$ and $y = 2$

19. $y = 4 - x$ when $x = 3$ and $y = 1$

20. $y = 2 - \frac{1}{2}x$ when $x = 1$ and $y = 2$

21. Ali's resting metabolic rate is 1,320 calories per day. She goes for a 6-mile run in the morning and burns calories at a rate of 100 calories per mile. After school she goes bowling. If she burns 3 calories per minute, approximately how many minutes will she need to bowl to burn a total of 2,000 calories for the day? Show all work necessary to justify your answer.

Tic-Tac-Toe ~ Parallel or Perpendicular Lines



Perpendicular lines are lines that intersect at a 90° angle. Parallel lines never intersect. Each pair of lines given below is either parallel or perpendicular.

SET #1	SET #2	SET #3	SET #4	SET #5
$y = 2x + 3$	$-3x + 2y = 6$	$y - x = 5$	$-x + 2y = -4$	$y = \frac{1}{3}x + 3$
$y = 2x - 4$	$y = -\frac{2}{3}x - 4$	$y = x - 2$	$4x + 2y = 8$	$y = -3x - 1$

Step 1: If necessary, convert each equation into slope-intercept form using the method shown in **Lesson 3.4.**

Step 2: Graph each pair of equations on the same coordinate plane.

Step 3: State whether each pair of lines is parallel or perpendicular.

Step 4: After completing all five sets of graphs, develop a hypothesis on how to use the slope-intercept equation to determine if lines are parallel or perpendicular without graphing. Explain how you arrived at your hypothesis.